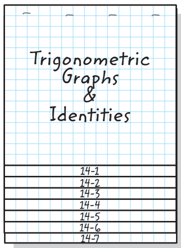
Chapter 14 Study Guide and Review

Download Vocabulary Review from algebra2.com

FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Graphing Trigonometric Functions (Lesson 14-1)

- For trigonometric functions of the form $y = a \sin b\theta$ and $y = a \cos b\theta$, the amplitude is $|a|$, and the period is $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$.
- The period of $y = a \tan b\theta$ is $\frac{180°}{|b|}$ or $\frac{\pi}{|b|}$.

Translations of Trigonometric Graphs (Lesson 14-2)

- For trigonometric functions of the form $y = a \sin (\theta - h) + k$, $y = a \cos (\theta - h) + k$, $y = a \tan (\theta - h) + k$, the phase shift is to the right when h is positive and to the left when h is negative. The vertical shift is up when k is positive and down when k is negative.

Trigonometric Identities (Lessons 14-3, 14-4, and 14-7)

- Trigonometric identities describe the relationships between trigonometric functions.
- Trigonometric identities can be used to simplify, verify, and solve trigonometric equations and expressions.

Sum and Difference of Angles Formulas (Lesson 14-5)

- For all values of α and β:
  $\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
  $\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

Double-Angle and Half-Angle Formulas (Lesson 14-6)

- Double-angle formulas:
  $\sin 2\theta = 2 \sin \theta \cos \theta$
  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
  $\cos 2\theta = 1 - 2 \sin^2 \theta$
  $\cos 2\theta = 2 \cos^2 \theta - 1$
- Half-angle formulas:
  $\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$
  $\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos \alpha}{2}}$

Key Vocabulary

amplitude (p. 823)
difference of angles formula (p. 849)
double-angle formula (p. 853)
half-angle formula (p. 855)
midline (p. 831)
phase shift (p. 829)
sum of angles formula (p. 849)
trigonometric equation (p. 861)
trigonometric identity (p. 837)
vertical shift (p. 831)

Vocabulary Check

Choose the correct term from the list above to compete each sentence.

1. The horizontal translation of a trigonometric function is a(n) __________.
2. A reference line about which a graph oscillates is a(n) __________.
3. The vertical translation of a trigonometric function is called a(n) __________.
4. The __________ formula can be used to find $\cos 22\frac{1}{2}°$.
5. The __________ can be used to find sin 60° using 30° as a reference.
6. The __________ can be used to find the sine or cosine of 75° if the sine and cosine of 45° and 30° are known.
7. A(n) __________ is an equation that is true for all values for which every expression in the equation is defined
8. The __________ can be used to find the sine or cosine of 65° if the sine and cosine of 90° and 25° are known.
9. The absolute value of half the difference between the maximum value and the minimum value of a periodic function is called the __________.

Vocabulary Review at algebra2.com

CHAPTER 14

Study Guide and Review

Lesson-by-Lesson Review

14–1 Graphing Trigonometric Functions (pp. 822–828)

Find the amplitude, if it exists, and period of each function. Then graph each function.

10. $y = -\frac{1}{2}\cos\theta$ **11.** $y = 4\sin 2\theta$

12. $y = \sin\frac{1}{2}\theta$ **13.** $y = 5\sec\theta$

14. $y = \frac{1}{2}\csc\frac{2}{3}\theta$ **15.** $y = \tan 4\theta$

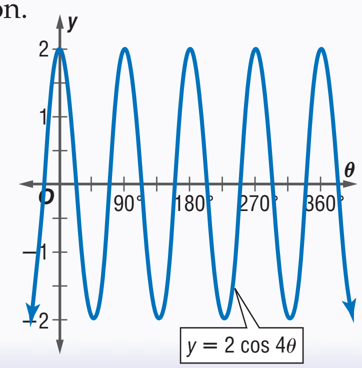
16. MECHANICS The position of a piston can be modeled using the equation $y = A\sin\left(\frac{1}{4} \cdot 2\pi t\right)$ where A is the amplitude of oscillation and t is the time in seconds. Determine the period of oscillation.

Example 1 Find the amplitude and period of $y = 2\cos 4\theta$. Then graph.

The amplitude is $|2|$ or 2.

The period is $\frac{360°}{|4|}$ or 90°.

Use the amplitude and period to graph the function.



14–2 Translations of Trigonometric Graphs (pp. 829–836)

State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.

17. $y = \frac{1}{2}\sin[2(\theta - 60°)] - 1$

18. $y = 2\tan\left[\frac{1}{4}(\theta - 90°)\right] + 3$

19. $y = 3\sec\left[\frac{1}{2}\left(\theta + \frac{\pi}{4}\right)\right] + 1$

20. $y = \frac{1}{3}\cos\left[\frac{1}{3}\left(\theta - \frac{2\pi}{3}\right)\right] - 2$

21. BIOLOGY The population of a species of bees varies periodically over the course of a year. The maximum population of bees occurs in March, and is 50,000. The minimum population of bees occurs in September and is 20,000. Assume that the population can be modeled using the sine function. Write an equation to represent the population of bees p, t months after January.

Example 2 State the vertical shift, amplitude, period, and phase shift of $y = 3\sin\left[2\left(\theta - \frac{\pi}{2}\right)\right] - 2$. Then graph the function.

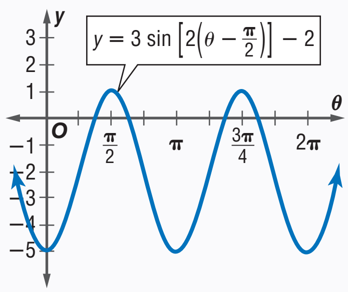
Identify the values of k, a, b, and h.

$k = -2$, so the vertical shift is –2.

$a = 3$, so the amplitude is 3.

$b = 2$, so the period is $\frac{2\pi}{|2|}$ or π.

$h = \frac{\pi}{2}$, so the phase shift is $\frac{\pi}{2}$ to the right.



14–3 Trigonometric Identities (pp. 837–841)

Find the value of each expression.

22. $\cot \theta$, if $\csc \theta = -\frac{5}{3}$; $270° < \theta < 360°$

23. $\sec \theta$, if $\sin \theta = \frac{1}{2}$; $0° \leq \theta < 90°$

Simplify each expression.

24. $\sin \theta \csc \theta - \cos^2 \theta$

25. $\cos^2 \theta \sec \theta \csc \theta$

26. $\cos \theta + \sin \theta \tan \theta$

27. $\sin \theta (1 + \cot^2 \theta)$

28. **PHYSICS** The magnetic force on a particle can be modeled by the equation $F = qvB \sin \theta$, where F is the magnetic force, q is the charge of the particle, B is the m agnetic field strength, and θ is the angle between the particle's path and the direction of the magnetic field. Write an equation for the magnetic force in terms of $\tan \theta$ and $\sec \theta$.

Example 3 **Find $\cos \theta$ if $\sin \theta = -\frac{3}{4}$ and $90° < \theta < 180°$.**

$\cos^2 \theta + \sin^2 \theta = 1$	Trigonometric identity
$\cos^2 \theta = 1 - \sin^2 \theta$	Subtract $\sin^2 \theta$ from each side.
$\cos^2 \theta = 1 - \left(\frac{3}{4}\right)^2$	Substitute $\frac{3}{4}$ for $\sin \theta$.
$\cos^2 \theta = 1 - \frac{9}{16}$	Square $\frac{3}{4}$.
$\cos^2 \theta = \frac{7}{16}$	Subtract.
$\cos \theta = \pm \frac{\sqrt{7}}{4}$	Take the square root of each side.

Since θ is in the second quadrant, $\cos \theta$ is negative. Thus, $\cos \theta = -\frac{\sqrt{7}}{4}$.

Example 4 **Simplify $\sin \theta \cot \theta \cos \theta$.**

$\sin \theta \cot \theta \cos \theta$

$= \frac{\sin \theta}{1} \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{1}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
$= \cos^2 \theta$	Multiply.

14–4 Verifying Trigonometric Identities (pp. 842–846)

Verify that each of the following is an identity.

29. $\frac{\sin \theta}{\tan \theta} + \frac{\cos \theta}{\cot \theta} = \cos \theta + \sin \theta$

30. $\frac{\sin \theta}{1 - \cos \theta} = \csc \theta + \cot \theta$

31. $\cot^2 \theta \sec^2 \theta = 1 + \cot^2 \theta$

32. $\sec \theta (\sec \theta - \cos \theta) = \tan^2 \theta$

33. **OPTICS** The amount of light passing through a polarization filter can be modeled using the equation $I = I_m \cos^2\theta$, where I is the amount of light passing through the filter, I_m is the amount of light shined on the filter, and θ is the angle of rotation between the light source and the filter. Verify the identity $I_m \cos^2\theta = I_m - \frac{I_m}{\cos^2\theta + 1}$.

Example 5 **Verify that $\tan \theta + \cot \theta = \sec \theta \csc \theta$ is an identity.**

$\tan \theta + \cot \theta \stackrel{?}{=} \sec \theta \csc \theta$	Original equation
$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \stackrel{?}{=} \sec \theta \csc \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cot \theta = \frac{\cos\theta}{\sin \theta}$
$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \stackrel{?}{=} \sec \theta \csc \theta$	Rewrite using the LCD, $\cos \theta \sin \theta$.
$\frac{1}{\cos \theta \sin \theta} \stackrel{?}{=} \sec \theta \csc \theta$	$\sin^2 \theta + \cos^2 \theta = 1$
$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \stackrel{?}{=} \sec \theta \csc \theta$	Rewrite as the product of two expressions.
$\sec \theta \csc \theta = \sec \theta \csc \theta$	$\frac{1}{\cos \theta} = \sec \theta$, $\frac{1}{\sin \theta} = \csc \theta$

CHAPTER 14

Study Guide and Review

14–5 Sum and Difference of Angles Formula (pp. 848–852)

Find the exact value of each expression.

34. $\cos 15°$ **35.** $\cos 285°$

36. $\sin 150°$ **37.** $\sin 195°$

38. $\cos (-210°)$ **39.** $\sin (-105°)$

Verify that each of the following is an identity.

40. $\cos (90° + \theta) = -\sin \theta$

41. $\sin (30° - \theta) = \cos (60° + \theta)$

42. $\sin (\theta + \pi) = -\sin \theta$

43. $-\cos \theta = (\cos \pi + \theta)$

Example 6 Find the exact value of $\sin 195°$.

$\sin 195° = \sin (150° + 45°)$ $195° = 150° + 45°$

$= \sin 150° \cos 45° + \cos 150° \sin 45°$ $\alpha = 150°, \beta = 45°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ Evaluate each expression.

$= \frac{\sqrt{2} - \sqrt{6}}{4}$ Simplify.

14–6 Double-Angle and Half-Angle Formulas (pp. 853–859)

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

44. $\sin \theta = \frac{1}{4}; 0° < \theta < 90°$

45. $\sin \theta = -\frac{5}{13}; 180° < \theta < 270°$

46. $\cos \theta = -\frac{5}{17}; 90° < \theta < 180°$

47. $\cos \theta = \frac{12}{13}; 270° < \theta < 360°$

Example 7 Verify that $\csc 2\theta = \frac{\sec \theta}{2 \sin \theta}$ is an identity.

$\csc 2\theta \stackrel{?}{=} \frac{\sec \theta}{2 \sin \theta}$ Original equation

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{\frac{1}{\cos \theta}}{2 \sin \theta}$ $\csc \theta = \frac{1}{\sin \theta}$, $\sec \theta = \frac{1}{\cos \theta}$

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{1}{2 \sin \theta \cos \theta}$ Simplify the complex fraction.

$\frac{1}{\sin 2\theta} = \frac{1}{\sin 2\theta}$ $2 \sin \theta \cos \theta = \sin 2\theta$

14–7 Solving Trigonometric Equations (pp. 861–866)

Find all solutions of each equation for the interval $0° \leq \theta < 360°$.

48. $2 \sin 2\theta = 1$

49. $\cos^2 \theta + \sin^2 \theta = 2 \cos \theta$ 0°

50. PRISMS The horizontal and vertical components of an oblique prism can be modeled using the equations $Z_x = P \cos \theta$ and $Z_y = P \sin \theta$ where Z_x is the horizontal component, Z_y is the vertical component, P is the power of the prism, and θ is the angle between the prism and the horizontal. For what values of θ will the vertical and horizontal components be equivalent?

Example 8 Solve $\sin 2\theta + \sin \theta = 0$ if $0° \leq \theta < 360°$.

$\sin 2\theta + \sin \theta = 0$ Original equation

$2 \sin \theta \cos \theta + \sin \theta = 0$ $\sin 2\theta = 2 \sin \theta \cos \theta$

$\sin \theta (2 \cos \theta + 1) = 0$ Factor.

$\sin \theta = 0$ or $2 \cos \theta + 1 = 0$

$\theta = 0°$ or $180°$ $\cos \theta = -\frac{1}{2}$

$\theta = 120°$ or $240°$

The solutions are 0°, 120°, 180°, and 240°.